A Synopsis
on a research proposal titled

Theory of Approximate Reasoning for Fuzzy Optimization

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Reg. No.: VB-2264 OF 2016-2017
Dt: 28 Nov, 2016

***Keywords**:* Fuzzy optimization, Fuzzy logic, Approximate reasoning.

During the past several years fuzzy logic has been extensively studied. Since it is close to human intuition, it is easy to learn as well as easy to apply. For this reason, it has been successfully applied to a wide variety of industrial processes and consumer products --- chemical reactors, cement kilns, elevator systems, air-conditioners, voice-controlled robot helicopters, auto-focusing cameras and many others.

Many decision-making tasks are too complex to be understood quantitatively, however, humans succeeded by using knowledge that is imprecise rather than precise. Fuzzy logic resembles human reasoning in its use of imprecise information to generate decisions. Unlike classical logic which requires a deep understanding of a system, exact equations, and precise numeric values, fuzzy logic incorporates an alternative way of thinking, which allows modeling complex systems using a higher level of abstraction originating from our knowledge and experience.

Fuzzy logic allows us to express knowledge with subjective concepts such as very big and a long time which are mapped into exact numeric ranges. Since knowledge can be expressed in a more natural way by using fuzzy sets, many decision (and engineering) problems can be greatly simplified.

Some essential characteristics of fuzzy logic relate to the following:

- In fuzzy logic, exact reasoning is viewed as a limiting case of approximate reasoning
- In fuzzy logic, everything is a matter of degree
- In fuzzy logic, knowledge is interpreted a collection of elastic or, equivalently, fuzzy constraint on a collection of variables
- Inference is viewed as a process of propagation of elastic constraints
- Any logical system can be fuzzified.

Zadeh first formulated the basic principles of **approximate reasoning** which is considered to be a framework to study the remarkable human ability to understand natural language. Research in artificial intelligence has shown that a great deal of commonsense reasoning is behind humans' intelligent behavior. It is also well established that most of commonsense reasoning is approximate in nature (presence of imprecise rule, insufficient and/or incomplete information). As commonsense knowledge is expressed in natural language such descriptions may not always of either black or white type and, hence, fall outside the domain of classical logic. This actually motivated Zadeh to consider fuzzy logic and a theory of fuzzy sets in dealing with problems of human reasoning.

There are two main characteristics of fuzzy systems that give them better performance for specific applications.

- Fuzzy systems are suitable for uncertain or approximate reasoning, especially for systems whose mathematical models are difficult to derive
- Fuzzy logic allows decision making with estimated values under incomplete or uncertain information.

Fuzzy set theory was applied to optimization problems and fuzzy optimization was introduced by Bellman and Zadeh in 1970. A non-iterative algorithm for fuzzy optimization was presented by Zimmermann in 1983 which was considered as a more flexible approach towards fuzzy optimization.

Fuzzy optimization can be considered as a two-stage process which includes --- i) aggregation of fuzzy conditions (criteria and constraints) by logical conjunction in order to receive decision fuzzy set and; ii) defuzzification of decision fuzzy set for determination of a crisp solution of the problem.

In its present form, fuzzy optimization was considered by Filev and Angelov as a special application of the theory of approximate reasoning. Using approximate reasoning a more flexible and realistic problem is formulated. In this research, an attempt is made to analyze optimization problems under uncertainty and show that fuzzy logic tools and approximate reasoning methodology in its entirety aims at finding an optimal solution to such class of problems.

The compositional rule of inference scheme with several relations (called Multiple Fuzzy Reasoning (MFR) Scheme) defined by Zadeh has the general form

Fact:	*X* has property *P*
Relation 1:	*X* and Y are in relation W_1
. . . . .	
Relation m:	*X* and *Y* are in relation W_m
Consequence:	*Y* has property *Q*

Accordingly, in this research, our plan is to consider the following topics in detail --- the theory of approximate reasoning; approximate reasoning in fuzzy system modeling; aggregation in fuzzy system modeling; multiple approximate reasoning schemes --- compositional rule of inference; properties of the compositional rule of inference; computation of the compositional rule of inference under different t-norms. Similarity in approximate reasoning; similarity based approximate reasoning; fuzzification and defuzzification suitable for similarity based approximate reasoning methodology; fuzzy linear programming problem --- solution using approximate reasoning; optimization with linguistic variables; multi-objective optimization with linguistic variables; applications of multi-objective optimization in management of organization and strategy formation.

We shall try to interpret fuzzy linear programming (FLPP) problems (where some or all coefficients can be fuzzy sets and the inequality relations between fuzzy sets can be given by a certain fuzzy relation) as multiple fuzzy reasoning schemes (MFR), where the antecedents of the scheme correspond to the constraints of the FLP problem and the fact of the scheme is the objective of the FLP problem.

Stability and sensitivity analysis becomes more and more attractive in the area of multiple objective mathematical programming. Existing research on this topic usually investigate the impact of parameter changes (in the right hand side or/and the objective functions or/and the 'A-matrix' or/and the domination structure) on the solution in various models of vector maximization problems, e.g. linear or nonlinear, static or dynamic. We show that the possibility distribution of the objectives of an multi-objective possibilistic linear program (MPLP) with (continuous) fuzzy number coefficients is stable under small changes in the membership function of the fuzzy parameters.

Suppose, we are given a mathematical programming problem in which the functional relationship between the decision variables and the objective function is not completely known. Our knowledge-base consists of a block of fuzzy if-then rules, where the antecedent parts of the rules contain some linguistic values of the decision variables, and the consequent part consists of a linguistic value of the objective function. We hope that use of approximate reasoning method to determine the crisp functional relationship between the objective function and the decision variables, could provide us with an effective way of finding the solution and solve the resulting (usually nonlinear) programming problem to find a fair optimal solution to the original fuzzy problem.

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